CBSE SAMPLE PAPER - 03

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. $\int e^x[\sec x + \log(\sec x + \tan x)]\,dx = ?$ **[1]**

   a) $e^x \log \tan x + C$ b) $e^x \log(\sec x + \tan x) + C$

   c) $e^x \sec x + C$ d) None of these

2. If a point R(4, y, z) lies on the line segment joining the points P(2, -3, 4) and Q(8, 0, 10), then the distance of R from the origin is: **[1]**

   a) $2\sqrt{14}$ b) 6

   c) $2\sqrt{21}$ d) $\sqrt{53}$

3. If $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}, \vec{a} = 0$, then **[1]**

   a) $\vec{b} + \vec{c} = \overrightarrow{0}$ b) none of these

   c) $\vec{b} = \vec{c}$ d) $\vec{b} = \overrightarrow{0}$

4. If A and B are two events and $A \neq \phi, B \neq \phi$, then **[1]**

   a) $P(\frac{A}{B})P(\frac{B}{A}) = 1$ b) $P(\frac{A}{B}) = \frac{P(A)}{P(B)}$

   c) $P(\frac{A}{B}) = \frac{P(A \cap B)}{P(B)}$ d) $P(\frac{A}{B}) = P(A) \cdot P(B)$

5. $\int \sqrt{e^x - 1}\,dx$ **[1]**

   a) $\frac{1}{2}(e^x - 1)^{1/2} + C$ b) $\frac{3}{2}(e^x - 1)^{3/2} + C$

   c) None of these d) $\frac{2}{3}(e^x - 1)^{3/2} + C$

6. Let A and B be independent events with P (A) = 0.3 and P(B) = 0.4. Find P(B|A). **[1]**

a) 0.5
b) 0.4
c) 0.2
d) 0.3

7. The area bounded by the curves $y=\sqrt{x}$, 2y + 3 = x and the x-axis in the first quadrant is **[1]**

a) 36
b) 18
c) 9
d) none of these

8. The angle between the straight lines $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$ is **[1]**

a) 45°
b) 60°
c) 30°
d) 90°

9. If $|\vec{a}|=3$ and $-1\leq k\leq 2$, then $|k\vec{a}|$ lies in the interval. **[1]**

a) [- 3, 6]
b) [3, 6]
c) [0, 6]
d) [1, 2]

10. The differential equation y$\frac{dy}{dx}$ + x = c represents: **[1]**

a) Family of ellipses
b) Family of hyperbolas
c) Family of parabolas
d) Family of circles

11. If the area cut off from a parabola by any double ordinate is k times the corresponding rectangle contained by that double ordinate and its distance from the vertex, then k is equal to **[1]**

a) $\frac{2}{3}$
b) 3
c) $\frac{1}{3}$
d) $\frac{3}{2}$

12. $\int\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ dx can be evaluated by the substitution **[1]**

a) x = cosθ
b) x = sec θ
c) x = sin θ
d) x = tan θ

13. f(x) $=\frac{x}{(x^2+1)}$ is increasing in **[1]**

a) None of these
b) $(-1,\infty)$
c) $(-\infty,-1)\cup(1,\infty)$
d) (-1, 1)

14. The number of all possible matrices of order 3×3 with each entry 0 or 1 is **[1]**

a) 81
b) none of these
c) 512
d) 18

15. The system AX = B of n equations in n unknowns has infinitely many solutions if **[1]**

a) if det.A = 0, (adj A)B$\neq$O
b) det. A$\neq$0
c) if det. A$\neq$ 0, (adjA)B$\neq$ O
d) if det. A = 0 , (adj A) B =O

16. The matrix $\begin{bmatrix}5&10&3\\-2&-4&6\\-1&-2&b\end{bmatrix}$ is a singular matrix, if the value of b is **[1]**

a) 3
b) Non-existent
c) -3
d) 0

17. Range of $\sin^{-1}x$ is [1]

a) None of these

b) $[0, \pi]$

c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

d) $\left[0, \frac{\pi}{2}\right]$

18. The general solution of the DE $\log\left(\frac{dy}{dx}\right) = (ax + by)$ is [1]

a) $\frac{-e^{-by}}{b} = \frac{e^{ax}}{a} + C$

b) none of these

c) $e^{ax} - e^{-by} = C$

d) $be^{ax} + ae^{by} = C$

19. **Assertion (A):** The absolute maximum value of the function $2x^3 - 24x$ in the interval [1, 3] is 89. [1]

Reason (R): The absolute maximum value of the function can be obtained from the value of the function at critical points and at boundary points.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Minor of an element of a determinant of order $n(n \geq 2)$ is a determinant of order n. [1]

Reason (R): If A is an invertible matrix of order 2, then $\det(A^{-1})$ is equal to $\frac{1}{|A|}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the value of $\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right]$ [2]

22. Find the general solution of the differential equation: $x\frac{dy}{dx} - y = x + 1$ [2]

23. For the matrices $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1} A^{-1}$ [2]

OR

Solve the system of linear equation, using matrix method $2x + 3y + 3z = 5$; $x - 2y + z = -4$; $3x - y - 2z = 3$

24. Write a unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$. [2]

25. A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared at least once? [2]

Section C

26. Integrate $\int x^2\, tan^{-1}x\, dx$ [3]

27. Find the particular solution of differential equation $\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x$, $x \neq 0$, given that y = 0, when $x = \frac{\pi}{2}$. [3]

OR

Solve the differential equation $x(1 + y^2)\,dx - y(1 + x^2)\,dy = 0$, given that y = 0, when x = 1

28. Find the value of λ so that the four points A, B, C and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}, -\hat{j} - \hat{k}, 3\hat{i} + \lambda\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$, respectively are coplanar. [3]

OR

ABCD is a parallelogram. If L and M are the mid-points of BC and DC respectively, then express $\overrightarrow{AL}$ and $\overrightarrow{AM}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{AD}$. Also, prove that $\overrightarrow{AL} + \overrightarrow{AM} = \frac{3}{2}\overrightarrow{AC}$.

29. Prove that: $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\left(\sin^{3/2} x+\cos^{3/2} x\right)} dx = \frac{\pi}{4}$ **[3]**

OR

Evaluate: $\int \frac{dx}{x(x^4-1)}$.

30. If x = ae^t (sin t + cos t) and y = ae^t (sin t – cos t), prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$. **[3]**

31. Using the method of integration, find the area of the region bounded by the lines: 2x + y = 4, 3x - 2y = 6 and x - 3y + 5 = 0. **[3]**

Section D

32. Solve the following linear programming problem graphically. Minimise $z = 3x + 5y$ subject to the constraints **[5]**

$x + 2y \geq 10$

$x + y \geq 6$

$3x + y \geq 8$

$\mathbf{x}, \mathbf{y} \geq \mathbf{0}$.

33. Let A = [-1, 1]. Then, discuss whether the following functions defined on A are one-one, onto or bijective: **[5]**

i. $f(x) = \frac{x}{2}$

ii. g(x) = |x|

iii. h(x) = x|x|

iv. k(x) = x^2

OR

Let R be a relation on $N \times N$, defined by (a, b) R (c, d) ⇔ a + d = b + c for all (a, b), (c, d) ∈ $N \times N$. Show that R is an equivalence relation.

34. Find the image of the point (5, 9, 3) in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ **[5]**

OR

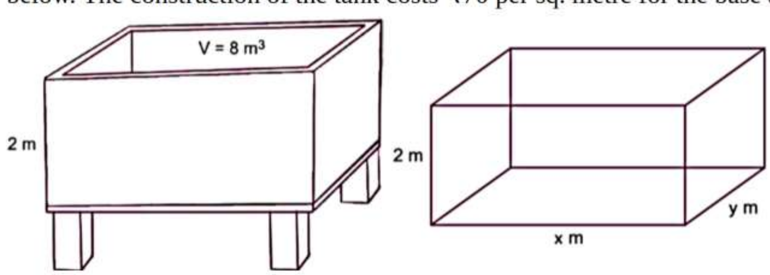
Find the coordinates of the point where the line through the points A (3, 4, 1) and B(5, 1, 6) crosses the XY-plane.

35. Show that the function $f(x) = \begin{cases} x^m \sin\left(\frac{1}{x}\right), x \neq 0 \\ 0, x = 0 \end{cases}$ is continuous but not differentiable at x = 0, if 0 < m < 1. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m^3 as shown below. The construction of the tank costs ₹70 per sq. metre for the base and ₹45 per square metre for sides.



(i) Express making cost C in terms of length of rectangle base.

(ii) If x and y represent the length and breadth of its rectangular base, then find the relation between the variables.

(iii) Find the value of **x** so that the cost of construction is minimum.

OR

Verify by second derivative test that cost is minimum at a critical point.

37. **Read the text carefully and answer the questions:** **[4]**

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats, and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold by school A, B, C are given below.

Article	School	A	B	C
Fans		40	25	35
Mats		50	40	50
Plates		20	30	40

(i) Represent the sale of handmade fans, mats and plates by three schools A, B and C and the sale prices (in ₹) of given products per unit, in matrix form.

(ii) Find the funds collected by school A, B and C by selling the given articles.

(iii) If they increase the cost price of each unit by 20%, then write the matrix representing new price.

OR

Find the total funds collected for the required purpose after 20% hike in price.

38. **Read the text carefully and answer the questions:** **[4]**

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) Find the probability that Ajay gets Grade A in all subjects.

(ii) Find the probability that he gets Grade A in no subjects.

Solution

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Class 12 - Mathematics

Section A

1. **(b)** e^x log (sec x + tan x) + C

   Explanation: I = $\int$ e^x {f(x) + f'(X)} dx, where f(x) = log (sec x + tan x)

   = e^x f(x) + C = e^x log (sec x + tan x) + C

2. **(a)** $2\sqrt{14}$

   Explanation: Here, P, Q, R are collinear

   $\therefore \overline{PR} = \lambda \overline{PQ}$

   $2\hat{i} + (y + 3)\hat{j} + (z - 4)\hat{k} = \lambda[6\hat{i} + 3\hat{j} + 6\hat{k}]$

   $\Rightarrow 6\lambda = 2$, y + 3 = 3λ, z - 4 = 6λ

   $\Rightarrow \lambda = \frac{1}{3}$, y = -2, z = 6

   $\therefore$ Point R(4, -2, 6)

   Now, OR = $\sqrt{(4)^2 + (-2)^2 + (6)^2} = \sqrt{56} = 2\sqrt{14}$

3. **(c)** $\vec{b} = \vec{c}$

   Explanation: $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$

   $\Rightarrow \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0$ and $\vec{a} \times \vec{b} - \vec{a} \times \vec{c} = 0$

   $\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$ and $\vec{a} \times (\vec{b} - \vec{c}) = 0$

   $\Rightarrow \vec{b} - \vec{c} = 0 \Rightarrow \vec{b} = \vec{c}$

   Also,

   $\Rightarrow |\vec{a}||\vec{b} - \vec{c}| \cos\theta = 0$ and $|\vec{a}||\vec{b} - \vec{c}| \sin\theta = 0$

   $\Rightarrow$ If $\theta = \frac{\pi}{2} \Rightarrow \sin\theta = 1 \Rightarrow \overrightarrow{\mathrm{b}} - \overrightarrow{\mathrm{c}} = 0 \Rightarrow \overrightarrow{\mathrm{b}} = \overrightarrow{\mathrm{c}}$

4. **(c)** $P(\frac{A}{B}) = \frac{P(A \cap B)}{P(B)}$

   Explanation: If $A \neq \phi$ and $B \neq \emptyset$, then by definition of conditional probability, $P(\frac{A}{B}) = \frac{P(A \cap B)}{P(B)}$

5. **(c)** None of these

   Explanation: Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

   Therefore ,

   $\Rightarrow \int \sqrt{e^x - 1} dx$

   Put e^x-1=t $\Rightarrow$ e^x dx=dt

   $\Rightarrow \int \sqrt{t} \frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t} dt$

   Put t=z^2 dt = 2z dz

   $\Rightarrow \int \frac{2z^2}{1+z^2} dz = \int \frac{2+2z^2-2}{1+z^2} dz = 2\int \frac{1+z^2}{1+z^2} dz - 2\int \frac{1}{1+z^2} dz$

   $= 2\int dz - 2\int \frac{1}{1+z^2} dz$ = 2 z-2 $\tan^{-1}$ z+c

   $= 2\sqrt{t} - 2\tan^{-1}\sqrt{t} + c \Rightarrow 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + c$

6. **(b)** 0.4

   Explanation: Let A and B be independent events with P (A) = 0.3 and P(B) = 0.4. P(B/A) = P(B) = 0.4

7. **(c)** 9

   Explanation: Required area:

   $$\int_0^9 \sqrt{x} dx - \int_3^9 \left(\frac{x-3}{2}\right) dx = \left[\frac{x^{\frac{3}{2}}}{3/2}\right]_0^9 - \frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^9 = 9 \text{ sq.units}$$

8. **(d)** 90°

   Explanation: We have,

   $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$

$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$

The direction ratios of the given lines are proportional to 2, 5, 4 and 1, 2, -3.

The given lines are parallel to the vectors $\overrightarrow{b_1} = 2\hat{i} + 5\hat{j} + 4\hat{k}$ and $\overrightarrow{b_2} = \hat{i} + 2\hat{j} - 3\hat{k}$

Let θ be the angle between the given lines.

Now,

$\cos\theta = \frac{\overrightarrow{b_1} \cdot \overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$

$= \frac{(2\hat{i}+5\hat{j}+4\hat{k})\cdot(\hat{i}+2\hat{j}-3\hat{k})}{\sqrt{2^2+5^2+4^2}\sqrt{1^2+2^2+(-3)^2}}$

$= \frac{2+10-12}{\sqrt{45}\sqrt{14}}$

= 0

$\Rightarrow \theta = 90^\circ$

9. **(c)** [0, 6]

Explanation: [0, 6] is the correct answer. The smallest value of $|k\vec{a}|$ will exist at numerically smallest value of k, i.e., at k = 0, which gives $|k\vec{a}| = |k||\vec{a}| = 0 \times 3 = 0$ The numerically greatest value of k is 2 at which $|k\vec{a}|$ = 6.

10. **(d)** Family of circles

Explanation: Given differential equation is

$y\frac{dy}{dx} + x = c$

$\Rightarrow$ ydy = (c - x)dx

On integrating both sides, we get

$\frac{y^2}{2} = cx - \frac{x^2}{2} + d$

$\Rightarrow$ $y^2 + x^2 - 2cx - 2d = 0$

Hence, it represents a family of circles whose centres are on the x-axis.

11. **(a)** $\frac{2}{3}$

Explanation: Required area :

$= 2\int_0^a \sqrt{4ax}dx$

$= k\alpha(2\sqrt{4a\alpha})$

$= \frac{8\sqrt{a}}{3}\alpha^{\frac{3}{2}}$

$= 4\sqrt{a}k\alpha^{\frac{3}{2}} \Rightarrow k = \frac{2}{3}$

12. **(d)** x = tan θ

Explanation: For $x = \tan\theta, \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right) = \tan^{-1}(\tan 3\theta) = 3\theta$

13. **(d)** (-1, 1)

Explanation: We have,$\Rightarrow$ $f(x) = \frac{x}{x^2+1}$

$\Rightarrow f'(x) = \frac{x^2-2x^2+1}{x^2+1}$

$\Rightarrow f'(x) = -\frac{x^2-1}{x^2+1}$

$\Rightarrow$ for critical points f'(x) = 0

when f'(x) = 0

We get x = 1 or x = -1

When we plot them on number line as f'(x) is multiplied by -ve sign we get

For x > 1 function is decreasing

For x < -1 function is decreasing

But between -1 to 1 function is increasing

$\therefore$ Function is increasing in (-1, 1)

14. **(c)** 512

Explanation: $2^{3x3} = 2^9 = 512$.

The number of elements in a 3 $\times$ 3 matrix is the product 3 $\times$ 3 = 9.

Each element can either be a 0 or a 1.

Given this, the total possible matrices that can be selected is $2^9 = 512$

15. **(d)** if det. A = 0 , (adj A) B =O

Explanation: If det. A = 0, (adj A) B =O⇒ The system AX = B of n equations in n unknowns may be consistent with infinitely many solutions or it may be inconsistent.

16. **(b)** Non-existent

Explanation: $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is singular matrix. So its determinant value of this matrix is zero.

i.e, $\begin{vmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{vmatrix} = 0$

$\Rightarrow 5(-4b + 12) - 10(-2b + 6) + 3(4 - 4) = 0$

$\Rightarrow -20b + 60 + 20b - 60 = 0$

b does not exist

17. **(c)** $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Explanation:

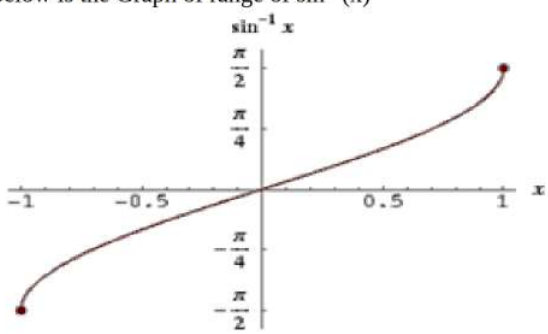
To Find: The range of $\sin^{-1}x$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \sin^{-1}(x)$ can be obtained from the graph of
Y = sin x by interchanging x and y axes.i.e, if (a, b) is a point on Y = sin x then (b, a) is

The point on the function $y = \sin^{-1}(x)$

Below is the Graph of range of $\sin^{-1}(x)$



From the graph, it is clear that the range $\sin^{-1}(x)$ is restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

18. **(a)** $\frac{-e^{-by}}{b} = \frac{e^{ax}}{a} + C$

Explanation: We have, $\log\left(\frac{dy}{dx}\right) = (ax + by)$

$\frac{dy}{dx} = e^{ax+by}$

$\frac{dy}{e^{by}} = e^{ax}dx$

On integrating on both sides, we obtain

$-\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + c$

19. **(d)** A is false but R is true.

Explanation: Let $f(x) = 2x^3 - 24x$

$\Rightarrow f'(x) = 6x^2 - 24 = 6(x^2 - 4)$

$= 6(x + 2)(x - 2)$

For maxima or minima put $f'(x) = 0$.

$\Rightarrow 6(x + 2)(x - 2) = 0$

$\Rightarrow x = 2, -2$

We first consider the interval [1, 3].

So, we have to evaluate the value of f at the critical point $x = 2 \in [1, 3]$ and at the end points of [1, 3].

At x = 1, $f(1) = 2 \times 1^3 - 24 \times 1 = -22$

At x = 2, $f(2) = 2 \times 2^3 - 24 \times 2 = -32$

At x = 3, f(3) = 2 × 3^3 - 24 × 3 = -18

∴ The absolute maximum value of f(x) in the interval [1, 3] is -18 occurring at x = 3.

Hence, Assertion is false and Reason is true.

20. **(d)** A is false but R is true.

Explanation: Assertion: Minor of an element of a determinant of order n(n ≥ 2) is a determinant of order n - 1.

So, Assertion is false.

Reason: We know, $AA^{-1} = I$

∴ $|AA^{-1}| = |I| \Rightarrow |A|\,|A^{-1}| = 1$

$\Rightarrow \left|A^{-1}\right| = \frac{1}{|A|}$

So, reason is true.

Section B

21. Let $\cot^{-1}\left(\frac{-5}{12}\right) = y$

Then $\cot y = \frac{-5}{12}$

Now,

$\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right] = \sin 2y$

$= 2\sin y\cos y = 2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)$ $\left[\text{since } \cot y < 0, \text{ so } y \in \left(\frac{\pi}{2}, \pi\right)\right]$

$= \frac{-120}{169}$

22. The given equation may be written as $\frac{dy}{dx} - \frac{1}{x}y = \frac{x+1}{x}$

This is of the form $\frac{dy}{dx} + Py = Q$, where $P = \frac{-1}{x}, Q = \frac{x+1}{x}$

$1F = e^{\int r\,dt} = e^{-\int \frac{1}{x}dx} = e^{-\log|x|} = \frac{1}{x}$

So, the required solution is given by $y \times 1.F = \int Q \times 1.F\,dx$

$\Rightarrow y \cdot e^{2x} = \int x \cdot e^{4x} \cdot e^{2x}\,dx$

$\Rightarrow y.\,e^{2x} = \int x \cdot e^{4x} \cdot e^{2x}dx \Rightarrow y \cdot e^{2x} = \int x \cdot e^{6x}dx \quad \Rightarrow y \cdot e^{2x} = x\int e^{6x}dx - \int\left[-\frac{dx}{dx}\int e^{3x}dx\right]dx$

$\Rightarrow ye^{2x} = x \cdot \frac{e^{6x}}{6} - \int\frac{e^{6x}}{6}dx \Rightarrow ye^{2x} = \frac{x}{6}e^{6x} - \frac{e^{6x}}{36} + c$

$\therefore y = \frac{xe^{4x}}{6} - \frac{e^{4x}}{36} + ce^{-2x}$.

23. Given: $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

$\Rightarrow |A| = 1$

Adj A = $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

and, B = $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

$|B| = -1$

$B^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{-1}\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$

Also, AB = $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 14 \\ 29 & 37 \end{bmatrix}$

|AB| = 407 – 406 = 1

And, adj(AB) = $\begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

$(AB)^{-1} = \frac{\text{adj AB}}{|\text{AB}|} = \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

Now, $B^{-1}A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$= \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

Hence, $(AB)^{-1} = B^{-1}A^{-1}$

OR

Matrix form of given equations is AX = B

$$\Rightarrow \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

Here $|A| = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$$

= 2(4 + 1) - 3(- 2 - 3) + 3(- 1 + 6)

$= 10 + 15 + 15 = 40 \neq 0$

Therefore, solution is unique and $X = A^{-1}B = \frac{1}{|A|}(adj.\, A)\, B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Therefore, x = 1, y = 2 and z = - 1 .

24. Given,

$\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$

$\therefore \vec{a} + \vec{b} = (2\hat{i} + 2\hat{j} - 5\hat{k}) + (2\hat{i} + \hat{j} - 7\hat{k}) = 4\hat{i} + 3\hat{j} - 12\hat{k}$

$\Rightarrow |\vec{a} + \vec{b}| = \sqrt{4^2 + 3^2 + (-12)^2} = \sqrt{16 + 9 + 144} = \sqrt{169} = 13$

$\therefore$ Required unit vector $\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \frac{4\hat{i}+3\hat{j}-12\hat{k}}{13} == \frac{4}{13}\hat{i} + \frac{3}{13}\hat{j} - \frac{12}{13}\hat{k}$

25. We know, when a pair of dice is thrown, total possible outcomes are = 36

A = getting sum as 8

= {(2,6),(3,5),(4,4),(5,3),(6,2)}

Therefore, P(A) = $\frac{5}{36}$

B = Getting number 5 atleast once = {(1,5),(2,5),(3,5),(4,5),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,5)}

Hence P(B) = $\frac{11}{36}$

(A ∩ B) = getting sum as 8 with at least one die showing 5 = {(3,5),(5,3)

Hence P(A ∩ B) = $\frac{2}{36}$

Therefore the required probability is given by, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{\frac{2}{36}}{\frac{5}{36}} = \frac{2}{5}$

Section C

26. Here, we take inverse trigonometric function as first function and algebraic function as second function.

Let I $= \int x^2\, tan^{-1}x dx$

$= \int (tan^{-1}x).\, x^2\, dx$

$= (tan^{-1}x).\, \frac{x^3}{3} - \int \frac{1}{1+x^2}.\, \frac{x^3}{3} dx$

$= \frac{x^3}{3} tan^{-1}x - \frac{1}{3} \int \frac{x^3}{x^2+1} dx$

$= \frac{x^3}{3} tan^{-1}x - \frac{1}{3} \int (x - \frac{x}{x^2+1}) dx$

$= \frac{x^3}{3} tan^{-1}x - \frac{1}{3} \int x dx + \frac{1}{6} \int \frac{2x}{x^2+1} dx$

$= \frac{x^3}{3} tan^{-1}x - \frac{1}{6}x^2 + \frac{1}{6} log(x^2 + 1)$

27. We have, $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x, (x \neq 0)$

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$.

Here, P = cot x and Q = 2x + x^2 cot x.

$\therefore \quad \mathrm{IF}=e^{\int \mathrm{Pdx}}=e^{\int \cot x d x} \;=e^{\log |\sin x|}=\sin x$

The general solution is given by

$y \cdot \mathrm{IF}=\int(\mathrm{IF} \times Q) d x+C$

$\Rightarrow y \cdot \sin x=\int\left(2 x+x^2 \cot x\right) \sin x d x+C$

$=2 \int x \sin x d x+\int x^2 \cos x d x+c$

$=2 \int x \sin x d x+x^2 \sin x-\int 2 x \sin x d x+C$

$\Rightarrow \quad y \cdot \sin x=x^2 \sin x+C$ ...(i)

On putting $x=\frac{\pi}{2}$ and y = 0 in Eq. (i), we get

$0 \cdot \sin \frac{\pi}{2}=\left(\frac{\pi}{2}\right)^2 \cdot \sin \frac{\pi}{2}+C \Rightarrow C=-\frac{\pi^2}{4}$

On putting $C=\frac{-\pi^2}{4}$ in Eq. (i), we get

$y \cdot \sin x=x^2 \sin x-\frac{\pi^2}{4}$

$\therefore \quad y=x^2-\frac{\pi^2}{4} cosec x$

[divding both sides by sin x]

OR

We have,

x (1 + y^2) dx - y (1 + x^2) dy = 0

$\Rightarrow$ x (1 + y^2) dx = y (1 + x^2) dy

$\Rightarrow \frac{x}{1+x^2} d x=\frac{y}{1+y^2} d y$

$\Rightarrow \frac{2 x}{1+x^2} d x=\frac{2 y}{1+y^2} d y$

Integrating both sides, we get

$\int \frac{2 x}{1+x^2} d x=\int \frac{2 y}{1+y^2} d y$

$\Rightarrow$ log |1 + x^2| = log |1 + y^2| + log C

$\Rightarrow$ log$\left|\frac{1+x^2}{1+y^2}\right|$ = log C

$\Rightarrow \frac{1+x^2}{1+y^2}$ = C

$\Rightarrow$ (1 + x^2) = (1 + y^2) C ...(i)

It is given that when x = 1, y = 0. So, putting x = 1 and y = 0 in (i), we get

(1+1) = (1 + 0) C $\Rightarrow$ C = 2

Putting C = 2 in (i), we get

(1 + x^2) = 2 (1 + y^2),

Which is the required solution.

28. According to the question,

Given, $\overrightarrow{O A}=4 \hat{i}+5 \hat{j}+\hat{k}$,

$\overrightarrow{O B}=-\hat{j}-\hat{k}$,

$\overrightarrow{O C}=3 \hat{i}+\lambda \hat{j}+4 \hat{k}$ and

$\overrightarrow{O D}=-4 \hat{i}+4 \hat{j}+4 \hat{k}$.

Now, $\overrightarrow{A B}=\overrightarrow{O B}-\overrightarrow{O A}=-\hat{j}-\hat{k}-(4 \hat{i}+5 \hat{j}+\hat{k})$

$=-4 \hat{i}-6 \hat{j}-2 \hat{k}$

$\overrightarrow{A C}=\overrightarrow{O C}-\overrightarrow{O A}=3 \hat{i}+\lambda \hat{j}+4 \hat{k}-(4 \hat{i}+5 \hat{j}+\hat{k})$

$=-\hat{i}+(\lambda-5) \hat{j}+3 \hat{k}$

and $\overrightarrow{A D}=\overrightarrow{O D}-\overrightarrow{O A}$

$=-4 \hat{i}+4 \hat{j}+4 \hat{k}-(4 \hat{i}+5 \hat{j}+\hat{k})$

$=-8 \hat{i}-\hat{j}+3 \hat{k}$

Since, vectors $\overrightarrow{O A}, \overrightarrow{O B}, \overrightarrow{O C}$ and $\overrightarrow{O D}$ are coplanar.

$\therefore \quad[\overrightarrow{A B}\ \overrightarrow{A C}\ \overrightarrow{A D}]=0$

$$\therefore \begin{vmatrix} -4 & -6 & -2 \\ -1 & (\lambda-5) & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$$

$-4(3\lambda - 15 + 3) + 6(-3 + 24) - 2(1 + 8\lambda - 40) = 0$

$\Rightarrow \quad -4(3\lambda - 12) + 6(21) - 2(8\lambda - 39) = 0$

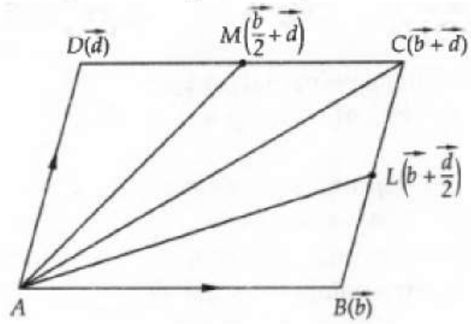
$\Rightarrow \quad -12\lambda + 48 + 126 - 16\lambda + 78 = 0$

$\Rightarrow \quad -28\lambda + 252 = 0$

$\Rightarrow \lambda = 9$

OR

Taking A as the origin, let the position vectors of vertices B and D of parallelogram ABCD be $\overrightarrow{b}$ and $\overrightarrow{d}$ respectively.



From figure,

$\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{AD} = \vec{d}$.

In triangle ABC, we have

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\Rightarrow \quad \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD}$ $[\because \overrightarrow{BC} = \overrightarrow{AD}]$

$\Rightarrow \quad \overrightarrow{AC} = \vec{b} + \vec{d}$

$\therefore$ Position vector of C is $\vec{b} + \vec{d}$

Since L and M are mid-points of BC and CD respectively. Therefore,

Position vector of $L = \frac{\vec{b} + (\vec{b} + \vec{d})}{2} = \vec{b} + \frac{1}{2}\vec{d}$, Position vector of M $= \frac{(\vec{b} + \vec{d}) + \vec{d}}{2} = \frac{\vec{b}}{2} + \vec{d}$

$\therefore \overrightarrow{AL}$ = Position vector of L - Position vector of A $= \vec{b} + \frac{1}{2}\vec{d} - \overrightarrow{0} = \vec{b} + \frac{1}{2}\vec{d} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{AD}$

and $\overrightarrow{AM} =$ Position vector of M - Position vector of A $= \frac{\vec{b}}{2} + \vec{d} - \overrightarrow{0} = \frac{\vec{b}}{2} + \vec{d} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{AD}$

$\therefore \overrightarrow{AL} + \overrightarrow{AM} = \left(\vec{b} + \frac{1}{2}\vec{d}\right) + \left(\frac{1}{2}\vec{b} + \vec{d}\right) = \frac{3}{2}\vec{b} + \frac{3}{2}\vec{d} = \frac{3}{2}(\vec{b} + \vec{d}) = \frac{3}{2}\overrightarrow{AC}$

29. Let the given integral be, $y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$

Using theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

$y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx$

$y = \int_0^{\pi/2} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$ .... (ii)

Adding equation(i) and equation.(ii) we get.

$2y = \int_0^{\pi/2} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$

$2y = \int_0^{\pi/2} 1dx$

$2y = (x)_0^{\frac{\pi}{2}}$

$y = \frac{\pi}{4}$

OR

Let the given integral be,

$I=\int \frac{dx}{x\left(x^4-1\right)} dx$

Putting t = x^4

dt = $4x^3$dx

$I=\int \frac{x^3 dx}{x^4\left(x^4-1\right)}=\frac{1}{4} \times \int \frac{dt}{t(t-1)}$

Now putting, $\frac{1}{t(t-1)}=\frac{A}{t}+\frac{B}{t-1}$ .... (1)

A(t - 1) + Bt = 1

Now put t - 1 = 0

Therefore, t = 1

A(0) + B(1) = 1

B = 1

Now put t = 0

A(0 - 1) + B(0) = 1

A = -1

Now From equation (1) we get,

$\frac{1}{t(t-1)}=\frac{-1}{t}+\frac{1}{t-1}$

$\therefore\ \frac{1}{4} \int \frac{1}{t(t-1)} dt=-\frac{1}{4} \int \frac{1}{t} dt+\frac{1}{4} \int \frac{1}{t-1} dt$

$=-\frac{1}{4} \log t+\frac{1}{4} \log |t-1|+c$

$=-\frac{1}{4} \log x^4+\frac{1}{4} \log \left|x^4-1\right|+c$

$=-\log |x|+\frac{1}{4} \log \left|x^4-1\right|+c$

30. x = ae^t (sin t + cos t) and y = ae^t (sin t - cos t) (Given)

$\therefore \frac{dx}{dt}$ = a[e^t(cos t - sin t) + e^t(sin t + cos t)]

= -y + x

and $\frac{dy}{dt}$ = a[e^t (sin t - cos t) + e^t(sin t + cos t)]

= y + x

Therefore

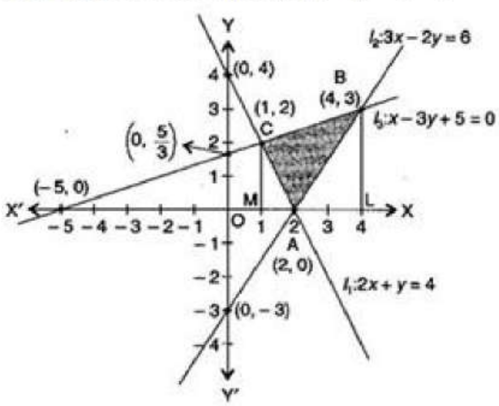
$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$

or $\frac{dy}{dx}=\frac{x+y}{x-y}$

31. Equation of one line l_1 is 2x + y = 4,

Equation of second line l_2 is 3x - 2y = 6

And Equation of third line l_3 is x - 3y + 5 = 0



Here, vertices of triangle ABC are A (2, 0), B (4, 3) and C (1, 2).

Now, Required area of triangle

= Area of trapezium CLMB - Area $\triangle ACM$ - Area $\triangle ABL$

$=\left|\int_1^4 \frac{1}{3}(x+5) dx\right|-\left|\int_1^2(4-2x) dx\right|-\left|\int_2^4 \frac{3}{2}(x-2) dx\right|$

$=\frac{1}{3}\left|\left(\frac{x^2}{2}+5x\right)_1^4\right|-\left|\left(4x-\frac{2x^2}{2}\right)_1^2\right|-\frac{3}{2}\left|\left(\frac{x^2}{2}-2x\right)_2^4\right|$

$=\frac{1}{3}\left[8+20-\left(\frac{1}{2}+5\right)\right]-\{(8-4)-(4-1)\}-\frac{3}{2}|(8-8)-(2-4)|$

$= \frac{1}{3}\left(28 - \frac{11}{2}\right) - (4 - 3) - \frac{3}{2} \times 2$

$= \frac{1}{3} \times \frac{45}{2} - 1 - 3$

$= \frac{15}{2} - 1 - 3 = \frac{7}{2}$ sq. units

Section D

32. We have

$x + 2y \geq 10$ .... (i)

$x + y \geq 6$ .... (ii)

$3x + y \geq 8$ .... (iii)

x, $y \geq 0$

First of all find point of interection by taking x + 2y = 10

x + y = 6

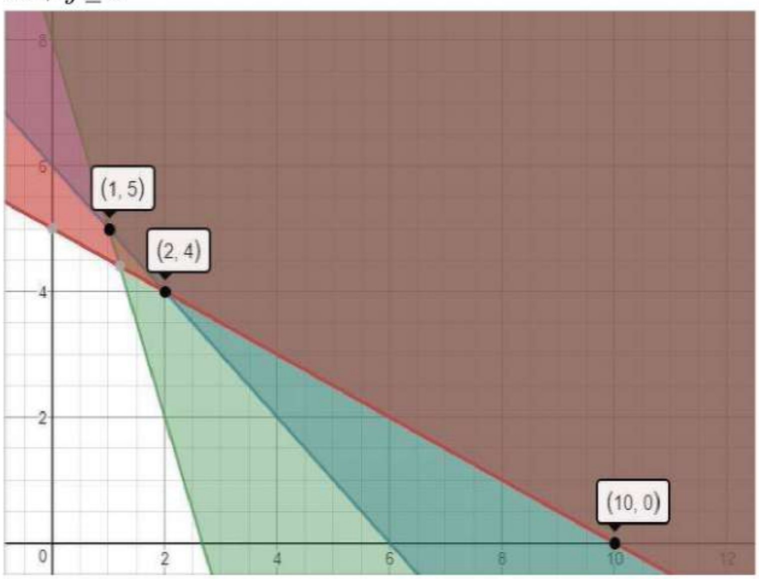
3x + y = 8

On solving these equation simultaneously you will get point of intersection.

$x + 2y \geq 10$

$x + y \geq 6$

$3x + y \geq 8$



Z = 3x + 5y

Put (1, 5)

Z = 3 + 25 = 28

Put (2, 4)

Z = 6 + 20 = 26

Hence minimum value of Z is 26.

33. Given that A = [-1, 1]

i. $f(x) = \frac{x}{2}$

Let $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{2} = \frac{x_2}{2} \Rightarrow x_1 = x_2$

So, f(x) is one-one.

Now, let $y = \frac{x}{2}$

$\Rightarrow x = 2y \notin A, \ \forall y \in A$

As for $y = 1 \in A, \ x = 2 \notin A$

So, f(x) is not onto.

Also, f(x) is not bijective as it is not onto.

ii. g(x) = |x|

Let $g(x_1) = g(x_2)$

$\Rightarrow |x_1| = |x_2| \Rightarrow x_1 = \pm x_2$

So, g(x) is not one-one.

Now, x=$\pm$y $\notin A$ for all y $\in$ R

So, g(x) is not onto, also, g(x) is not bijective.

iii. h(x) = x|x|

$\Rightarrow x_1|x_1| = x_2|x_2| \Rightarrow x_1 = x_2$

So, h(x) is one-one

Now, let y = x|x|

$\Rightarrow y = x^2 \in A, \forall x \in A$

So, h(x) is onto also, h(x) is a bijective.

iv. k(x) = x^2

Let $k(x_1) = k(x_2)$

$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

Thus, k(x) is not one-one.

Now, let y = x^2

$\Rightarrow x\sqrt{y} \notin A, \forall y \in A\ x = \sqrt{y} \notin A, \forall y \in A$

As for y = -1, $x = \sqrt{-1} \notin A$

Hence, k(x) is neither one-one nor onto.

OR

Here R is a relation on $N \times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N \times N$

We shall show that R satisfies the following properties

i. Reflexivity:

We know that a + b = b + a for all a, b $\in$ N.

$\therefore$ (a, b) R (a, b) for all (a, b) $\in (N \times N)$

So, R is reflexive.

ii. Symmetry:

Let (a, b) R (c, d). Then,

(a, b) R (c, d) $\Rightarrow$ a + d = b + c

$\Rightarrow$ c + b = d + a

$\Rightarrow$ (c, d) R (a, b).

$\therefore$ (a, b) R (c, d) $\Rightarrow$ (c, d) R (a, b) for all (a, b), (c, d) $\in N \times N$

This shows that R is symmetric.

iii. Transitivity:

Let (a, b) R (c, d) and (c, d) R (e, f). Then,

(a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow$ a + d = b + c and c + f = d + e

$\Rightarrow$ a + d + c + f = b + c + d + e

$\Rightarrow$ a + f = b + e

$\Rightarrow$ (a, b) R (e, f).

Thus, (a, b) R (c, d) and (c, d) R (e, f) $\Rightarrow$ (a, b) R (e, f)

This shows that R is transitive.

$\therefore$ R is reflexive, symmetric and transitive

Hence, R is an equivalence relation on $N \times N$

34. Suppose,

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

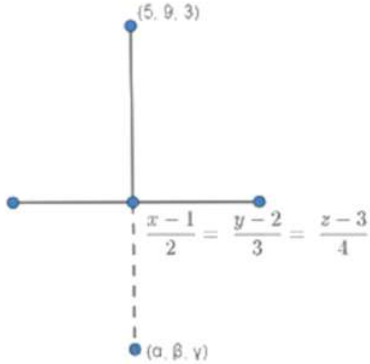
So the foot of the perpendicular is $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

The direction ratios of the perpendicular is

$(2\lambda + 1 - 5) : (3\lambda + 2 - 9) : (4\lambda + 3 - 3)$

$\Rightarrow (2\lambda - 4) : (3\lambda - 7) : (4\lambda)$

Direction ratio of the line is 2 : 3 : 4



From the direction ratio of the line and the direct ratio of its perpendicular, we have

$2(2\lambda-4)+3(3\lambda-7)+4(4\lambda)=0$

$\Rightarrow 4\lambda-8+9\lambda-21+16\lambda=0$

$\Rightarrow 29\lambda=29$

$\Rightarrow \lambda=1$

Therefore, the foot of the perpendicular is (3, 5, 7)

The foot of the perpendicular is the mid-point of the line joining (5, 9, 3) and (α,β,γ)

Therefore, we have

$\frac{\alpha+5}{2}=3\Rightarrow\alpha=1$

$\frac{\beta+9}{2}=5\Rightarrow\beta=1$

$\frac{y+3}{2}=7\Rightarrow\gamma=11$

Therefore, the image is (1, 1, 11)

OR

The vector equation of the line through point A and B is

$\vec{r}=3\hat{i}+4\hat{j}+\hat{k}+\lambda\left[(5-3)\hat{i}+(1-4)\hat{j}+(6-1)\hat{k}\right]$

$\Rightarrow\vec{r}=3\hat{i}+4\hat{j}+\hat{k}+\lambda\left(2\hat{i}-3\hat{j}+5\hat{k}\right)$

$\vec{r}=(3+2\lambda)\hat{i}+(4-3\lambda)\hat{j}+(1+5\lambda)\hat{k}$ .....(1)

Let P be the point where the line AB crosses the XY plane. Then the position vector $\vec{r}$ of the point P is the form $x\hat{i}+y\hat{j}$

Then, from (1), we have,

$x\hat{i}+y\hat{j}=(3+2\lambda)\hat{i}+(4-3\lambda)\hat{j}+(1+5\lambda)\hat{k}$

$\Rightarrow x=3+2\lambda, y=4-3\lambda$, $1+5\lambda=0$

Now, $1+5\lambda$ gives, $\lambda=-\frac{1}{5}$

$\therefore x=3+2(\frac{-1}{5})$ and $y=4-3(\frac{-1}{5})$

$\Rightarrow x=\frac{13}{5}$ and $y=\frac{23}{5}$

Hence the required point is $\left(\frac{13}{5},\frac{23}{5},0\right)$

35. LHL = $\lim\limits_{x\to0^-}f(x)$

= $\lim\limits_{h\to0}f(0-h)$

= $\lim\limits_{h\to0}(-h)^m\sin\left(-\frac{1}{h}\right)$

= $\lim\limits_{h\to0}-(-h)^m\sin\left(\frac{1}{h}\right)$

= 0 × k [when $-1\le k\le1$]

= 0

RHL = $\lim\limits_{x\to0^+}f(x)$

= $\lim\limits_{h\to0}f(0+h)$

= $\lim\limits_{h\to0}(0+h)^m\sin\left(\frac{1}{0+h}\right)$

= $\lim\limits_{h\to0}(h)^m\sin\left(\frac{1}{h}\right)$

= 0 x k [when $-1\le k'\le1$]

= 0

LHL = RHL = f(0)

Since, f(x) is continuous at x = 0

For Differentiability at x = 0

(LHD at x = 0) = $\lim\limits_{x \to 0^-} \frac{f(x)-f(0)}{x-0}$

$= \lim\limits_{h \to 0} \frac{f(0-h)-f(0)}{(0-h)-0}$

$= \lim\limits_{h \to 0^-} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{h}$

$= \lim\limits_{h \to 0^-} -(-h)^{m-1} \sin\left(\frac{1}{h}\right)$

= Not Defined [since 0<m<1]

(RHD at x = 0) = $\lim\limits_{x \to 0^+} \frac{f(x)-f(0)}{x-0}$

$= \lim\limits_{h \to 0^+} \frac{f(0+h)-f(0)}{(0+h)-0}$

$= \lim\limits_{h \to 0^+} \frac{h^m \sin\left(\frac{1}{h}\right)}{0+h-0}$

$= \lim\limits_{h \to 0^+} (h)^{m-1} \sin\left(\frac{1}{h}\right)$

= 0

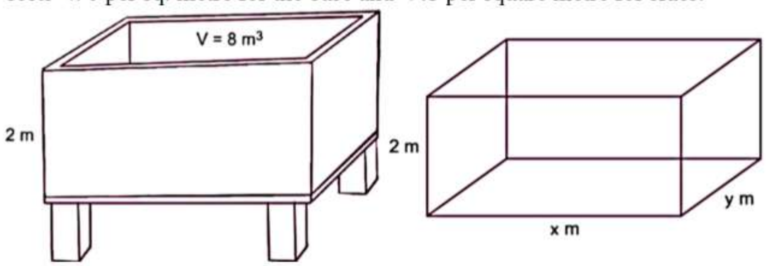
Since , (LHD at x = 0)$\neq$(RHD at x = 0)

Hence, f(x) is continuous at x = 0 but not differentiable.

Section E

36. **Read the text carefully and answer the questions:**

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m^3 as shown below. The construction of the tank costs ₹70 per sq. metre for the base and ₹45 per square metre for sides.



(i) Since 'C' is cost of making tank

$\therefore$ C = 70xy + 45 $\times$ 2(2x + 2y)

$\Rightarrow$ C = 70xy + 90(2x + 2y)

$\Rightarrow$ C = 70xy + 180(x + y) [$\because 2 \cdot x \cdot y = 8 \Rightarrow y = \frac{8}{2x} \Rightarrow y = \frac{4}{x}$]

$\Rightarrow$ C = 70x $\times \frac{4}{x} + 180\left(x + \frac{4}{x}\right)$

$\Rightarrow$ C = 280 + 180$\left(x + \frac{4}{x}\right)$

(ii) x$\cdot$y = 4

Volume of tank = length $\times$ breadth $\times$ height (Depth)

8 = $x \cdot y \cdot$ 2

$\Rightarrow$ 2xy = 8 $\Rightarrow$ xy = 4

(iii)For maximum or minimum

$\frac{dC}{dx}$ = 0

$\frac{d}{dx}$(280 + 180(x + $\frac{4}{x}$)) = 0 $\Rightarrow$ $180\left(1 + 4\left(-\frac{1}{x^2}\right)\right)$ = 0

$\Rightarrow$ $180\left(1 - \frac{4}{x^2}\right)$ = 0 $\Rightarrow 1 - \frac{4}{x^2}$ = 0

$\Rightarrow \frac{4}{x^2}$ = 1 $\Rightarrow$ x^2 = 4

$\Rightarrow$ x = $\pm$2

$\Rightarrow$ x = 2 (length can never be negative)

OR

Now, $\frac{d^2C}{dx^2} = 180\left(+\frac{8}{x^3}\right)$

$\Rightarrow \frac{d^2C}{dx^2}\Big|_{x=2} = 180 \times \frac{8}{8} = 180 = +ve$

Hence, to minimize C, x = 2m

37. **Read the text carefully and answer the questions:**

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats, and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold by school A, B, C are given below.

Article	School	A	B	C
Fans		40	25	35
Mats		50	40	50
Plates		20	30	40

(i)

$$P = \begin{matrix} & Fans & Mats & Plates \\ A & 40 & 50 & 20 \\ B & 25 & 40 & 30 \\ C & 35 & 50 & 40 \end{matrix}$$

$$Q = \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} Fans \\ Mats \\ Plates \end{matrix}$$

(ii) Clearly, total funds collected by each school is given by the matrix

$$PQ = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 1000 + 5000 + 1000 \\ 625 + 4000 + 1500 \\ 875 + 5000 + 2000 \end{bmatrix} = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$$

$\therefore$ Funds collected by school A is ₹7000.

Funds collected by school B is ₹6125.

Funds collected by school C is ₹7875.

(iii)

$$New\ \ price\ matrix\ Q = 20\% \times \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} Fans \\ Mats \\ Plates \end{matrix}$$

$$\Rightarrow Q = \begin{bmatrix} 25 + 25 \times 0.20 \\ 100 + 100 \times 0.20 \\ 50 + 50 \times 0.20 \end{bmatrix} \begin{matrix} Fans \\ Mats \\ Plates \end{matrix}$$

$$Q = \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix} \begin{matrix} Fans \\ Mats \\ Plates \end{matrix}$$

New price matrix Q = $20\% \times \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$

$\Rightarrow$ Q = $\begin{bmatrix} 25 + 25 \times 0.20 \\ 100 + 100 \times 0.20 \\ 50 + 50 \times 0.20 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$

Q = $\begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$

OR

$$PQ = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix}$$

$$PQ = \begin{bmatrix} 1200 + 6000 + 1200 \\ 750 + 4800 + 1800 \\ 1050 + 6000 + 2400 \end{bmatrix} = \begin{bmatrix} 8400 \\ 7350 \\ 9450 \end{bmatrix}$$

Total fund collected = 8400 + 7350 + 9450 = ₹25,200

38. **Read the text carefully and answer the questions:**

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) P(Grade A in Maths) = P(M) = 0.2

P(Grade A in Physics) = P(P) = 0.3

P(Grade A in Chemistry) = P(C) = 0.5

P(Grade A in all subjects) = $P(M \cap P \cap C) = P(M) \cdot P(P) \cdot P(C)$

P(Grade A in all subjects) = $0.2 \times 0.3 \times 0.5 = 0.03$

(ii) P(Grade A in Maths) = P(M) = 0.2

P(Grade A in Physics) = P(P) = 0.3

P(Grade A in Chemistry) = P(C) = 0.5

P(Grade A in no subjects) = $P(\bar{M} \cap \bar{P} \cap \bar{C}) = P(\bar{M}) \cdot P(\bar{P}) \cdot P(\bar{C})$

P(Grade A in no subjects) = $0.8 \times 0.7 \times 0.5 = 0.280$